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ASSESSING URBAN RAIL TRANSIT SYSTEMS VULNERABILITY: METRICS VS INTERDICTION MODELS

Stefano Starita[a], **Annunziata Esposito Amideo**[b], and Maria Paola Scaparra[b]

[a] *Warwick Business School, University of Warwick, Coventry, UK*

[b] *Kent Business School, University of Kent, Canterbury, UK*

SUMMARY

- Research Background
- Vulnerability Metrics
  - Connectivity-driven metrics
  - Path length-driven metrics
  - Flow-driven metrics
- Interdiction Models
  - The Path Interdiction Problem (PIP)
  - The Flow Interdiction Problem (FIP)
- London Underground: A Case Study
- Conclusions

RESEARCH BACKGROUND

Railway infrastructures have been repeatedly affected by disasters, either natural or man-made

Examples in the UK

- **Tube disruption**: coordinated suicide bomb attacks (CBBC Newsround 2015)
- **Euston Station closure**: a fire triggering a power cut (The Telegraph 2017)
- **London Bridge Rail and Tube disruption**: preventive measure due to a security alert (Evening Standard 2017)

RESEARCH BACKGROUND

Emerging issues

What are the most critical elements of the system whose disruption would significantly degrade the system's normal functioning?

Approaches

- Vulnerability Metrics
- Interdiction Models

VULNERABILITY METRICS

Aim of Vulnerability Metrics

To devise a ranking of the most critical network components, which can then be exploited to prioritize mitigation strategies

Examples of Vulnerability Metrics

- Maximal Flow (Murray 2013)
- Shortest Path (Murray 2013)
- Connectivity (Murray 2013)
- System Flow (Murray 2013)
- Network Importance (Balijepalli and Oppong 2014; Jenelius, Petersen, and Mattsson 2006)
- Robustness (Balijepalli and Oppong 2014; Scott et al. 2006)

VULNERABILITY METRICS

Connectivity-driven metrics

- **Node Degree (ND)**
- **Network Accessibility (NA)** (Ouyang et al. 2014):

$$NA = \frac{1}{n(n-1)} \sum_{i=1}^{n} n_{disr}^{i}$$

where n is the number of network nodes (stations) and n_{disr}^{i} is the number of nodes that can be reached from node i after an attack

Path length-driven metrics

- **Node Betweenness (NB)**
- **Network Topological Efficiency (E)** (Sum, Zhao, and Lu 2015):

$$E = \frac{1}{n(n-1)} \sum_{s,d=1}^{n} \frac{1}{SP_{sd}}$$

where SP_{sd} is the length of the shortest path connecting nodes s and d

- **Node Vulnerability (NV)**:

$$NV(i) = E(o) - E'(i)$$

Flow-driven metrics

- **Passenger Flow Influence (PFI)** (Sum, Zhao, and Lu 2015):

$$PFI = \sum \left(OF_i + DF_i + IF_i \right)$$

where OF_i, DF_i, and IF_i are the generated, attracted and intercepted flow for node i, respectively

INTERDICTION MODELS

Aim of Interdiction Models

To identify the most critical network components, the ones whose disruption would inflict the most serious damage to the system

Interdiction Models Applications

- Military purposes (Fulkerson 1977; Wollmer 1964)
- Service and supply chain systems (Church, Scaparra, and Middleton 2004)
- Network connectivity and cohesiveness (Addis, Di Summa, and Grosso 2013; Arulselvan et al. 2009; Granata, Steeger, and Rebennack 2013)

Detailed Surveys on Interdiction Models (Esposito Amideo and Scaparra 2017; Sullivan, Aultman-Hall, and Novak 2009)

INTERDICTION MODELS

The Path Interdiction Problem (PIP)

$$\max z = \sum_{s \in N} \sum_{d \in N} \left(\alpha Z_{sd} + (1-\alpha) K_{sd} Y_{sd} \right)$$

Bi-objective function

s.t.

$$\sum_{i \in N} X_i \leq D$$

Maximum *D* Nodes are Disrupted

$$Z_{sd} \leq \sum_{i \in N(p)} X_i \qquad \forall\ s,d \in N,\ p \in P(sd)$$

Path Disruption

$$Y_{sd} \leq (1 - Z_{sd}) LP_{sd} \qquad \forall\ s,d \in N$$

Path/Connectivity Link

$$Y_{sd} \leq \sum_{i \in N(p)} \beta_{sd} X_i + l_p \qquad \forall\ s,d \in N,\ p \in P(sd)$$

Path threshold

$$\left.\begin{aligned} &Z_{sd} \in \{0,1\} && \forall\ s,d \in N \\ &Y_{sd} \geq 0 && \forall\ s,d \in N \\ &X_i \in \{0,1\} && \forall\ i \in N \end{aligned}\right\} \text{Variable Domains}$$

INTERDICTION MODELS

<u>The Flow Interdiction Problem (FIP)</u>

(Matisziw and Murray 2009)

$$max\ z = \sum_{s \in N} \sum_{d \in N} f_{sd} Z_{sd}$$

Total Disrupted Flow Maximization

s.t.

$$\sum_{i \in N} X_i \leq D$$

Maximum *D* Nodes are Disrupted

$$Z_{sd} \leq \sum_{i \in N(p)} X_i \qquad \forall\ s,d \in N,\ p \in P(sd)$$

Path Disruption

$$Z_{sd} \in \{0,1\} \qquad \forall\ s,d \in N$$

$$X_i \in \{0,1\} \qquad \forall\ i \in N$$

Binary Variables

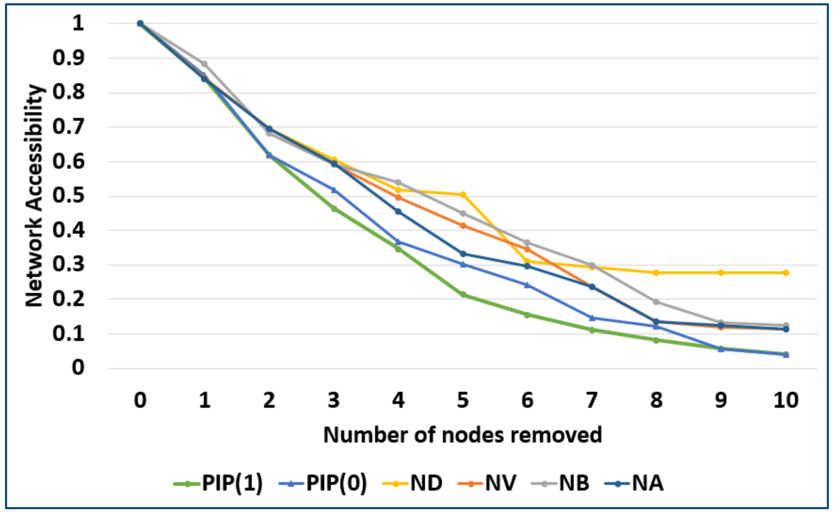
LONDON UNDERGROUND: A CASE STUDY

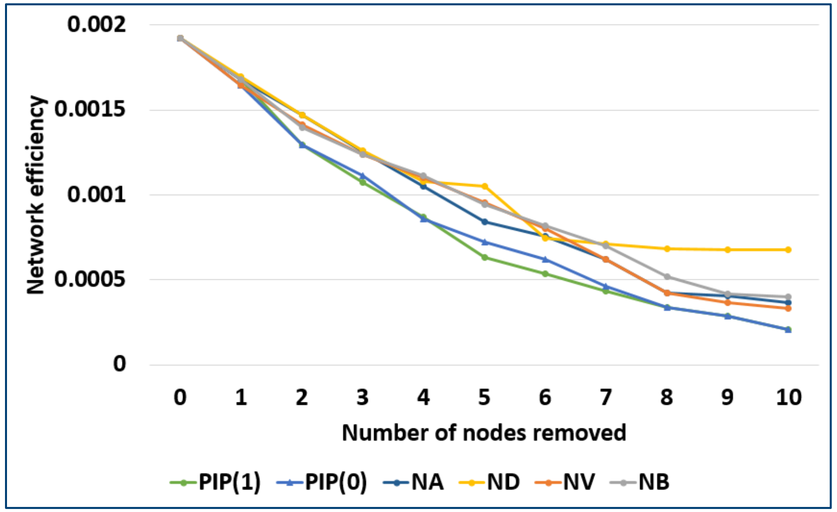
Network Data:

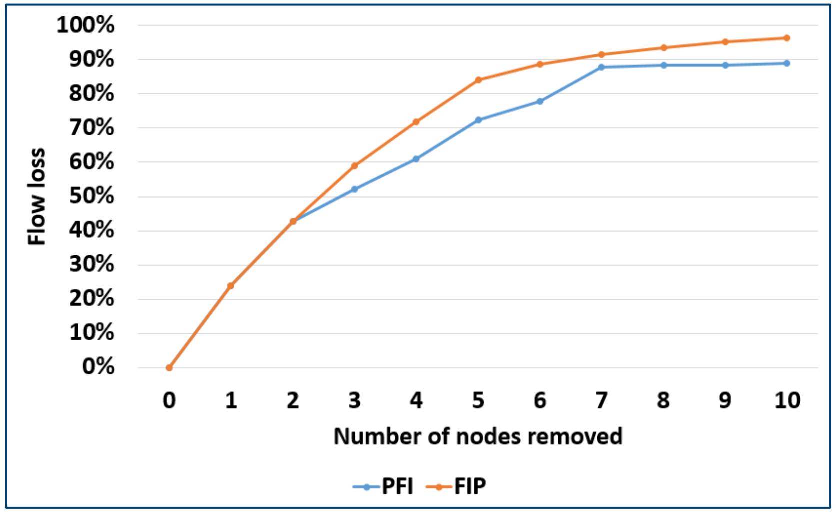
- 51 nodes (stations)
- 178 directed arcs (links)
- 10 metro lines

LONDON UNDERGROUND: A CASE STUDY

Static Ranking vs. Optimization/1







LONDON UNDERGROUND: A CASE STUDY

Static Ranking vs. Optimization/2

Metrics Comparison

NA	ND	NV	NB	PFI
Baker Street	**St Pancras**	**Green Park**	**Oxford Circus**	BankMonument
St Pancras	**Baker Street**	Bond Street	Bond Street	**Oxford Circus**
Green Park	**Oxford Circus**	**Oxford Circus**	**Green Park**	**St Pancras**
Bond Street	Embankment	**Baker Street**	Tottenham Court Road	**Green Park**
BankMonument	Farringdon	**St Pancras**	Holborn	Waterloo
Oxford Circus	**Green Park**	Holborn	**Baker Street**	Victoria
Holborn	Edgware Road	BankMonument	**St Pancras**	**Baker Street**
Embankment	Barbican	Embankment	BankMonument	Bond Street
Marble Arch	Euston Square	Tottenham Court Road	Westminster	Euston
Warren Street	Great Portland Street	Leicester Square	Marble Arch	Warren Street

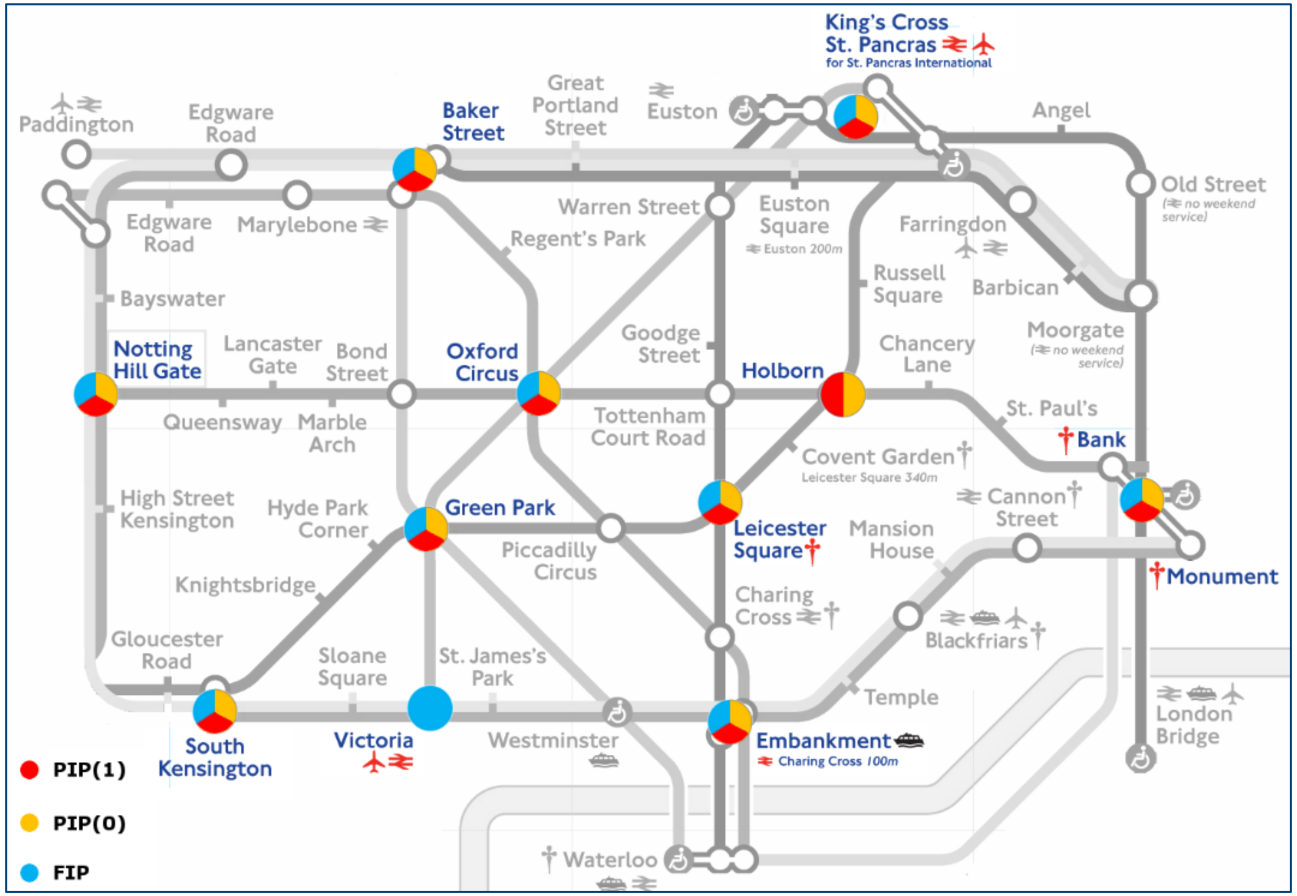
Table 1. Ten most critical stations for each metric

Models Comparison

PIP(1)		PIP(0)		FIP	
Station	Disr.	Station	Disr.	Station	Disr.
Green Park	9	**Green Park**	10	**BankMonument**	10
Oxford Circus	9	**Oxford Circus**	9	**St Pancras**	8
St Pancras	8	**BankMonument**	8	**Embankment**	7
BankMonument	7	**Embankment**	7	**Green Park**	7
Embankment	6	Holborn	6	**Oxford Circus**	7
Baker Street	5	**Notting Hill Gate**	5	**Leicester Square**	5
Notting Hill Gate	5	**Baker Street**	4	**Notting Hill Gate**	3
Holborn	3	**St Pancras**	3	Victoria	3
South Kensington	2	**South Kensington**	2	**Baker Street**	2
Leicester Square	1	**Leicester Square**	1	**South Kensington**	2

Table 2. Ten most frequently disrupted stations for each optimization model

LONDON UNDERGROUND: A CASE STUDY



The Ten Most Frequently Disrupted Stations for each Interdiction Model

CONCLUSIONS

This contribution investigated two alternative approaches to evaluate urban rail transit systems vulnerability:

- Vulnerability Metrics
- Interdiction Models

Vulnerability metrics tend to underestimate the real impact of disruptive events due to their inability to capture system components' interaction hence, **interdiction models** are more reliable tools

Further research directions:

- Disruption impact on other performance criteria (e.g. cohesiveness (Veremyev et al. 2014))
- Vulnerability assessment of other infrastructures (e.g., road networks, energy grids)
- Bi-level programs to devise effective protection strategies

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ANY QUESTIONS?